Journal of Applied Mechanics and Technical Physics, Vol. 45, No. 1, pp. 112–117, 2004

EXTENSION OF MIXED-MODE FATIGUE CRACKS IN STEEL SAMPLES

V. M. Tikhomirov and P. G. Surovin

UDC 620.178.6

Experimental studies of the extension of mixed-mode fatigue cracks were performed on rail steel samples. The direction of growth of fatigue cracks was studied for three types of loading: alternate transverse shear and transverse and longitudinal shears under compressing stresses. For all tested samples, the following common feature in the extension of fatigue cracks was established: the direction of crack growth coincides with the propagation direction of the principal stresses having minimum absolute values.

Key words: *fatigue crack, transverse shear, longitudinal shear.*

In failure mechanics, three basic types cracks are distinguished according to the type of loading: 1) opening mode (mode I) crack; 2) sliding mode (mode II) crack; and 3) tearing mode (mode III) crack. In contrast to mode I cracks, the mechanisms of extension of mixed mode II and III fatigue cracks have been studied insufficiently. It is known that the extension direction of such cracks does not coincide with the direction the original notch. There are a variety of the so-called local criteria for determining this direction based on an asymptotic distribution of stresses around the tip of a crack or an acute notch [1, 2]. Experimental studies of the subcritical growth and further unstable extension of mixed-mode cracks confirmed the validity of the proposed criteria for brittle materials (Plexiglas and glass) [3].

The direction of extension of mixed-mode cracks in constructional materials (steel and aluminum alloys) under cyclic loading differs from the extension direction in the case of slow crack growth under static forces [4, 5]. However, nearly all known experiments were performed under constant-sign loading. The extension of fatigue cracks in the zone of compressing rated stresses has also been studied inadequately.

In real structures, for example during interaction of a wheel and a railway rail, primarily alternate cyclic loads are observed and fatigue cracks develop predominantly in a zone of compressing stresses. In the present study using rail steel samples, we explored the growth of fatigue cracks of three types: 1) transverse shear under alternate loading; 2) transverse shear in a zone of rated compressing stresses; 3) longitudinal shear in a zone of rated compressing stresses.

1. Direction of Growth of Elliptic Cracks. The direction of crack growth from defects with nonzero apex curvature radius is determined by the stress state near the apex. We consider the problem of complex loading of an infinite plate with an elliptic opening (Fig. 1) which models a crack whose tip is subjected to plastic deformation.

We use the well-known solution of the problem of a tensile stress p applied at infinity to a plate with an elliptic notch whose major axis is inclined to the axis of tension at an angle β [6]. The circumferential stresses σ_θ on the notch contour are defined by the relation

$$\sigma_\theta = p\frac{1 - \cos 2(\theta - \beta) + m^2 + 2m\cos 2\beta}{1 - 2m\cos 2\theta + m^2}.$$

Here $m = (a - b)/(a + b)$, θ is the polar coordinate of the ellipse contour in its conformal mapping onto the circumference, and a and b are the major and minor semiaxes of the ellipse.

Siberian State University of Transport Communications, Novosibirsk 630049. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 45, No. 1, pp. 135–142, January–February, 2004. Original article submitted April 18, 2003.

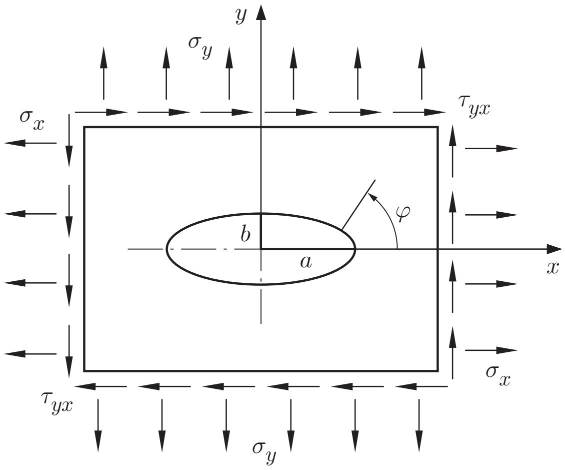

Fig. 1. Loading diagram of a plate with an elliptic opening.

To construct a solution of the problem considered, we sum the solution for the following four cases of loading: 1) $p = \tau_{yx}$ and $\beta = 45°$; 2) $p = -\tau_{yx}$ and $\beta = -45°$; 3) $p = \sigma_y$ and $\beta = 90°$; 4) $p = \sigma_x$ and $\beta = 0$. As a result, we obtain

$$\sigma_\theta = \frac{\sigma_y(1 + 2\cos 2\theta - m^2 - 2m) + \sigma_x(1 - 2\cos 2\theta - m^2 + 2m) - 4\tau_{yx}\sin 2\theta}{1 - 2m\cos 2\theta + m^2}. \tag{1}$$

We assume that failure begins at the point of the notch contour at which the circumferential stresses reach a peak value. From expression (1), we obtain the following condition to determine the position of the peak σ_θ:

$$\sin 2\theta[\sigma_x(1 - m^2) - \sigma_y(1 + m)^2](1 - m) - 2\tau_{yx}[\cos 2\theta(1 + m^2) - 2m] = 0. \tag{2}$$

Assuming that the crack grows along the normal to the notch contour, the direction of crack start (angle φ) can be determined from the following dependence given in [7]:

$$\varphi = \arctan\Big(\frac{1+m}{1-m}\tan\theta\Big). \tag{3}$$

Expressing $\cos 2\theta$ and $\sin 2\theta$ in Eq. (2) in terms of $\tan\theta$, we obtain

$$\tan\theta = \frac{1-m}{1+m}\,\frac{\sigma_y(1+m) - \sigma_x(1-m) - \sqrt{[\sigma_y(1+m) - \sigma_x(1-m)]^2 + 4\tau_{yx}^2}}{2\tau_{yx}}. \tag{4}$$

Finally, we have

$$\tan\varphi = \frac{\sigma_y(1+m) - \sigma_x(1-m) - \sqrt{[\sigma_y(1+m) - \sigma_x(1-m)]^2 + 4\tau_{yx}^2}}{2\tau_{yx}}. \tag{5}$$

From expression (4), it follows that for the case $m = 1$ where the ellipse degenerates into a crack, formula (3) contains an indeterminate form of the type of zero by zero, which can be evaluated using the asymptotic solution of [1].

We consider the case of an acute notch where $m \approx 1$. Relation (5) implies that the magnitude of the angle φ is not influenced by the stress σ_x acting along the major axis of the ellipse. In this case, if the plate is extended in the direction β by stresses at infinity, then, substituting the relations $\sigma_x = p\cos^2\beta$, $\sigma_y = p\sin^2\beta$, and $\tau_{yx} = p\sin\beta\cos\beta$ into Eq. (5), we obtain

$$\varphi = (\beta - \pi/2)/2, \qquad \beta \neq 0. \tag{6}$$

A similar relation was obtained in [3].

For pure shear, the direction of crack growth is determined from formula (5) by setting $\sigma_y = 0$ and $\sigma_x = 0$. In this case, the angle $\varphi = -45°$ for any $m \neq 1$.

The results of experiments obtained for static tension of polymethyl methacrylate based glass plates with oblique rectilinear notches [8] are not fitted by solution (6). The experimental data are satisfactorily fitted by an

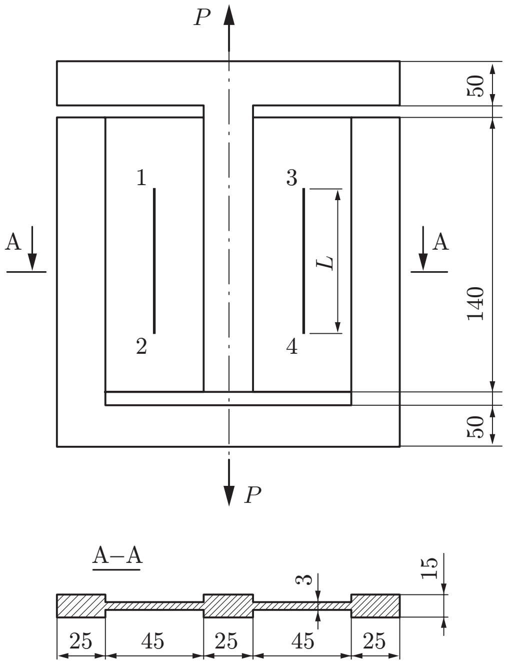

Fig. 2. Geometry of a sample of the 1st type.

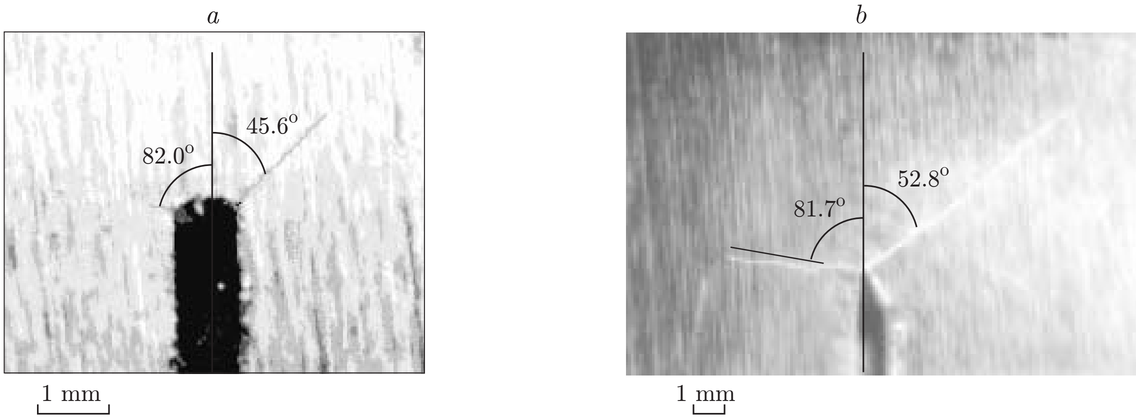

Fig. 3. Fatigue cracks extending from the notch apex (concentrator 1): a) crack in sample 1 after 53 thousand loading cycles ($L = 45$ mm); b) crack in sample 2 after 100 thousand loading cycles ($L = 60$ mm).

asymptotic solution with an additional term that allows for the influence of the stress σ_x [8] and also using the gradient criteria of failure [6].

2. Extension of Mixed-Mode Fatigue Cracks under Alternate Loading. The growth of mixed-mode cracks was examined on samples whose geometry is shown in Fig. 2. The plate had two symmetric notches of width 0.8 mm and length $L = 45$ and 60 mm. Cyclic loading led to extension of fatigue cracks from four concentrators (Fig. 2). The samples were previously stretched by a force of 50 kN using a special device, and cyclic loading ($P_{\max} = -20$ kN, $P_{\min} = -80$ kN) was then performed on a GPM-1 hydraulic pulsator. The real loading of the samples measured by four symmetrically located strain gauges was $P_{\max} = 30$ kN and $P_{\min} = -30$ kN (symmetric loading cycle).

The direction and length of fatigue cracks extending from four concentrators were examined with a microscope at different stages of the tests. Figure 3a shows a fatigue crack near the apex of a notch (concentrator 1 in Fig. 2) after 53 thousand loading cycles. It is evident that two fatigue cracks extend from the notch apex. The curvature radius at the concentrator apex is 0.5 mm.

TABLE 1

Sample number	Loading parameters		φ, deg		ψ, deg
	$\Delta\sigma$, MPa	$\Delta\tau$, MPa	Experimental data [5]	on [4]	
1	113.5	25.3	12.0	10.3 (14.2)	12.0 (0)
2	86.0	49.7	24.6	20.1 (18.3)	24.57 (0.1)
3	31.5	67.9	38.5	34.3 (10.9)	3.,5 (0)
4	0	62.8	45.0	45.0 (0)	45.0 (0)

Note. The difference (in percent) between numerical and experimental data is given in parentheses.

In another experiment, the curvature radius at the notch apex was 0.15 mm and the length of the notch was $L = 60$ mm. In this sample, the direction of fatigue crack extension did not change (Fig. 3b). With the same loading parameters, the crack growth rate was about three times higher than that in first sample.

The results of numerical finite-element calculations of the stress–strain states of the tested plate show that fatigue cracks originate in the zones of the highest concentration of tensile stresses. In the first half of the loading cycle, where the plate is stretched, the normal to the trajectory of the peak principal stress σ_1 coincides with the direction of growth of the left-hand fatigue crack (see Fig. 3), and in the second half of the cycle, where the sample is compressed, it coincides with the direction of extension of the right-hand crack. Thus, fatigue cracks extend in those directions in which shear is absent.

Chingshen [4] assumes that fatigue cracks grow in the direction of the so-called crack tip displacement vector. This assumption implies that the angle φ varies from 0 (opening mode crack) to 45° (pure shear crack). These conclusions are in conflict with the experimental data obtained in our study.

Table 1 presents experimental data on the direction of fatigue crack extension in thin-wall pipes with acute notches under simultaneous cyclic tension and torsion [5], values of the angle φ calculated using the criteria adopted in [4], and values of the angle ψ between the direction of rated stresses σ_3 in these samples and the major semiaxis of the notch calculated from the formula

$$\tan 2\psi = 2\Delta\tau/\Delta\sigma$$

($\Delta\sigma$ and $\Delta\tau$ are the stress amplitudes for tension and torsion of the samples, respectively).

The results given in Table 1 suggest that the mixed-mode fatigue cracks extend in the direction of rated principal stresses σ_3. It should be noted that for pure shear ($\Delta\sigma = 0$), the value of the angle φ differs only slightly from the values obtained using formula (5).

3. Extension of Sliding Mode Fatigue Cracks under Compressing Stresses. The incubation and extension of sliding mode fatigue cracks under compressing stresses were studied in tests with a $100 \times 100 \times 8$ mm steel plate. At the center of the plate made of rail steel there was an end-to-end notch 40 mm long at an angle of 43° to the loading direction (Fig. 4). The notch simulated a sliding mode crack with the sides subjected to compression.

The tests were performed with the following loading parameters: $P_{\max} = -20$ kN, $P_{\min} = -100$ kN and a loading frequency of 600 cycle/min (10 Hz). After 180 thousand cycles, fatigue cracks were observed which extended from the notch apexes at an angle of 42.9° to the original path of the notch. In what follows, the crack propagated in the same direction, and after 480 thousand cycles, they reached dimensions of 1.0–1.6 mm (Fig. 4). As the amplitude of the loading cycle increased by a factor of 1.56, the direction of crack extension did not change, and after 200 thousand loading cycles, the length of the fatigue crack increased by 2 mm. Thus, the rate of crack growth increased by a factor of almost three.

Numerical calculations using a finite-element model of this sample showed that fatigue cracks grow in a direction perpendicular to the direction of the minimum principal stresses σ_3 at the concentrator apex. Figure 5 gives the distribution of principal stresses under a compression load of 100 kN. Near the notch apex, the concentration of compressing stresses is almost 2.5 times higher than the concentration of tensile stresses. Fatigue cracks originate in the zone of peak compressing stresses, in contrast to the case considered in Sec. 2.

Investigation of the sample fracture after failure showed that the fatigue crack propagated in one plane. The crack front turned out to be concave. Thus, in the zone of compressing stresses, sliding mode fatigue cracks begin

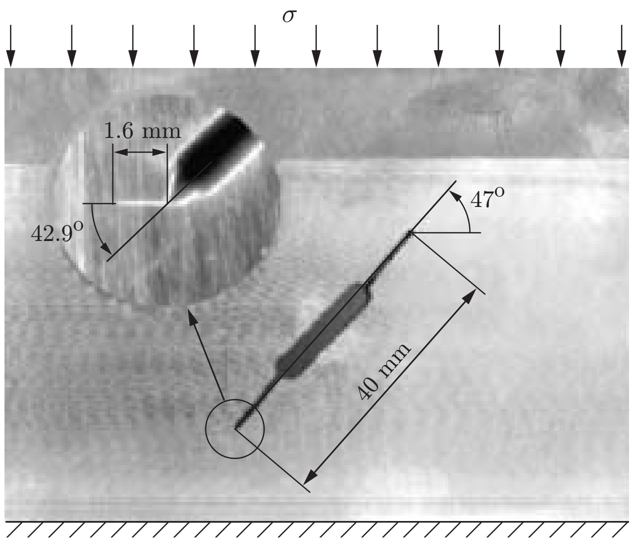

Fig. 4. Fatigue crack after 480 thousand loading cycles.

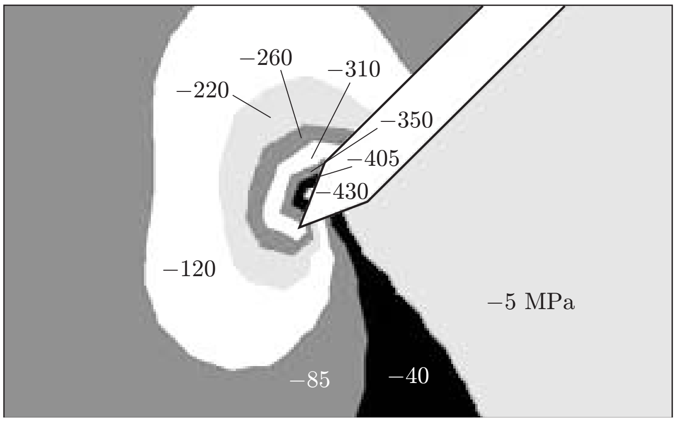

Fig. 5. Distribution of minimum principal stresses σ_3.

moving from the free surface of the sample, in contrast to opening mode fatigue cracks, which begin moving in the median plane of the plate [1].

4. Extension of Tearing Mode Fatigue Cracks under Compressing Stresses. Tearing mode fatigue cracks were studied on a sample which differed from that considered in Sec. 3 only in the type of concentrator. At each end of the plate ($100 \times 100 \times 8$ mm), two acute notches were made at an angle of 45° in the plane of the plate (Fig. 6). A notch 20 mm deep simulated a tearing mode crack with compressed sides.

Figure 7 shows the fracture of the sample after 200 thousand loading cycles ($P_{\max} = -20$ kN and $P_{\min} = -100$ kN). On one side, the fatigue crack grew by 6–7 mm, and on the other side, it grew by on 3–4 mm.

Investigation of the surface of the fatigue crack after sample failure showed that at the initial stage, a number of cracks originate, which are oriented at an angle of 50–30° to the plane of the sample and propagate in different directions. In the process of motion, the cracks merge and change orientation and direction of growth. Finally, the fatigue crack front becomes planar and moves in a direction perpendicular to the direction of application of the load.

5. Conclusions. The results of the studies lead to the following conclusions.

During growth, mixed-mode cracks tend to orient in the stress field so that shear of the crack sides is absent.

When the concentrations of tensile and compressing stresses are similar (samples of the 1st type), fatigue cracks grow in a direction perpendicular to the path of the peak principal stresses.

When the concentration of compressing stresses is higher than that of tensile stresses, a fatigue crack extends in a direction perpendicular to the path of the peak principal stresses (samples of the 2nd and 3rd types).

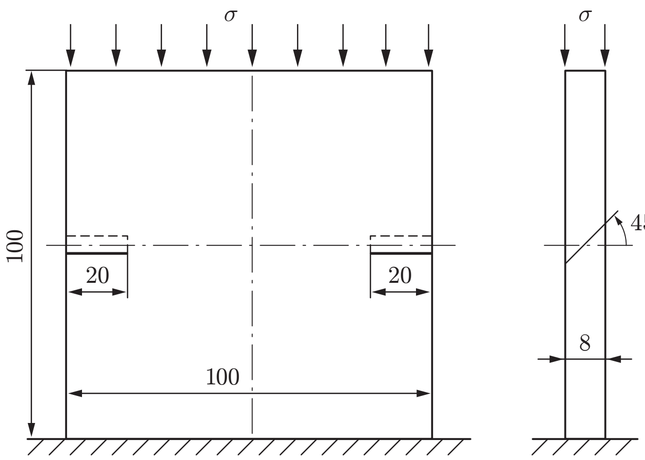

Fig. 6

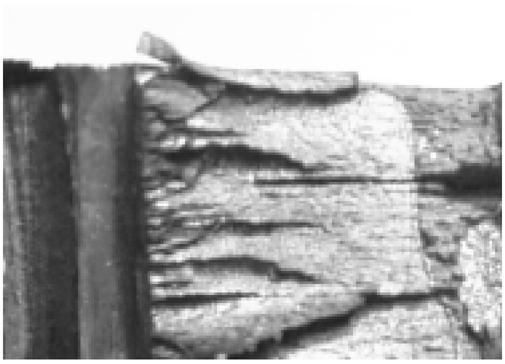

Fig. 7. Tearing mode fatigue crack.

In the cases studied, fatigue cracks ultimately propagated in a direction tangential to the path of principal stresses with minimum absolute values (σ_3 or σ_1).

In samples with acute notches in a homogeneous stress field, fatigue cracks extend primarily in one direction, which, even at the moment of start, does not coincide with the direction determined from asymptotic local criteria.

In a field of compressing cyclic stresses, fatigue cracks begin moving from the points on the free surface of the sample at which a plane stressed state occurs.

REFERENCES

1. V. Z. Parton and E. M. Morozov, *Mechanics of Elastoplastic Failure* [in Russian], Nauka, Moscow (1985).
2. V. N. Shlyannikov, "Mixed modes of crack extension in a complex stressed state (Review)," *Zavod. Lab.*, **56**, No. 6, 77–90 (1990).
3. F. Erdogan and G. C. Sih, "On the crack extension in plates under plane loading and transverse shear," *Trans. ASME, J. Basic Eng.*, No. 85, 519–527 (1963).
4. Li Chingshen, "Vector CTD criteria applied to mixed mode fatigue crack growth," *Fatigue Fract. Mater. Struct.*, **12**, No. 1, 59–65 (1989).
5. A. T. Yokobori, T. Yokobori, K. Sato, and K. Syoji, "Fatigue crack growth under mixed mode I and II," *Fatigue Fract. Mater. Struct.*, **8**, No. 8, 315–325 (1985).
6. N. I. Muskhelishvili, *Some Basic Problems of the Mathematical Theory of Elasticity*, Noordhoff, Leyden (1975).
7. M. A. Legan, "Determination of the breaking load and the position and direction of a fracture using the gradient approach," *J. Appl. Mech. Tech. Phys.*, **35**, No. 5, 750–756 (1994).
8. J. G. Williams and P. D. Ewing, "Crack under complex stress — the angled crack problems," *Int. J. Fract. Mech.*, **8**, No. 4, 441–446 (1972).